NEW MEXICO STATE UNIVERSITY  
THE KLIPSCH SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING  
Ph.D. QUALIFYING EXAMINATION  

August 15, 2011  
9:00 AM - 1:00 PM  
CLOSED BOOK  

Exam Instructions:  

a. Write the last four digits of your Banner ID number on the top of every page.  

b. Work six (6) problems from the three (3) areas of specialization selected at the time of registration. Do not work more than two (2) problems in any one area.  

A Circuits and Electronics  
B Communications  
C Computers  
D Control Systems  
E Digital Signal Processing  
F Electric Energy Systems  
G Electromagnetics  
H Photonics  

c. Check the boxes below indicating which six (6) problems you want graded. (You must work two problems from each of the three areas you specified at the time of registration.)  

<table>
<thead>
<tr>
<th></th>
<th>a</th>
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<th>c</th>
<th>d</th>
<th>e</th>
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<td>Photonics.......................</td>
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Design a fully differential two stage operational amplifier for a minimum gain-bandwidth product $GB = 100\text{MHz}$ and a load capacitance $C_L = 10\text{pF}$. Assume following parameters

0.13 $\mu$m CMOS technology: $k_n = 340\mu\text{A/V}^2$, $V_{thn} = 0.40\text{V}$, $k_p = 50\mu\text{A/V}^2$, $V_{thp} = 0.40\text{V}$, $\lambda_n = \lambda_p = 0.25\text{V}^{-1}$, $C_{ox} = 10\text{fF/}\mu\text{m}^2$ and a capacitive load $C_L = 10\text{pF}$ ($k_n = \mu C_{ox}/2$). Determine all transistor sizes and cascode biasing voltages including those in the biasing branch and in the common mode feedback network. Assume a single supply voltage $V_{DD} = 1.2\text{V}$ The design requires to determine bias currents, $W/L$ sizes for all transistors and compensation capacitance and resistance. Estimate for your design: a) location of the dominant pole and second (high frequency) pole taking into account only gate-source ($C_{gs}$) parasitic capacitances (do not take into account $C_{gd}$, $C_{bd}$ and $C_{sb}$ parasitic capacitances) the open loop gain, the slew rate, the common mode input range and the maximum output swing of the op-amp.
Design a fully differential folded cascode amplifier for a minimum gain-bandwidth product $GB=50\text{MHz}$ and a load capacitance $C_L=10\text{pF}$. Assume following 0.18$\mu$m CMOS technology technology parameters: $k_n=170\mu\text{A/V}^2$, $V_{thn}=0.45\text{V}$, $k_p=25\mu\text{A/V}^2$, $V_{thp}=0.45\text{V}$, $\lambda_n=\lambda_p=0.1\text{V}^{-1}$, $C_{ox}=4.2\text{fF/}\mu\text{m}^2$ ($kn=\mu C_{ox}/2$). Show the schema of the circuit including the common mode feedback network. Determine all transistor sizes including those in the biasing branch. Assume a single supply voltage $V_{DD}=1.8\text{V}$ The design requires to determine bias currents, W/L size all transistors, and to determine the value of all cascade biasing voltages. Estimate for your design: a) location of the dominant pole and second (high frequency) pole taking into account only gate-source ($C_{gs}$) parasitic capacitances (do not take into account $C_{gd}$, $C_{bd}$ and $C_{sb}$ parasitic capacitances) the open loop gain and the slew rate of the op-amp. Determine the Bandwidth, the common mode input range and the output impedance in unity gain voltage follower configuration.
Design a voltage follower with minimum bandwidth BW=50MHz in 0.13µm CMOS technology with kn_n=340µA/V^2, Vth_n=0.40V, kn_p=50µA/V^2, Vth_p=0.40V, λ_n=λ_p=0.25V^{-1}, C_{ox}=10fF/µm^2 and a capacitive load C_L=50pF (kn=µC_{ox}/2), v=0.4v^{1/2}. Determine biasing currents, all transistor sizes including those in the biasing branch. Assume a single supply voltage V_{DD}=1.2V, a load capacitance C_L=50pF and source resistance R_S=200kΩ. Estimate gain (assuming body effect is present), input and output poles and input/output voltage swing.
(i) The joint pdf of the random variables $X$ and $Y$ is given by

$$f_{XY}(x, y) = \begin{cases} 
1 & \text{in the two triangular regions shown in Figure 1} \\
0 & \text{otherwise}
\end{cases}$$

Let $Z = X + Y$. Find the pdf of $Z$.

(ii) A Gaussian random variable, with a mean 10 and a variance 100, is sampled repeatedly until a value between 10 and 12 is obtained, at which point the chance experiment ends. Let $N$ be the trial number upon which the experiment ends. Find the probability that $N > 5$.  
[Useful values: $Q(0.2)=0.421$]
(i) Find the expected value $E[X]$ and the correlation matrix $R_{XX}$ of a 2-dimensional random vector $X = [X_1 \ X_2]^T$ with a pdf

$$f_X(x) = \begin{cases} 2 & 0 \leq x_1 \leq x_2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

The notation $(\cdot)^T$ denotes transpose of a matrix.

(ii) Let $X(t)$ and $Y(t)$ be independent, wide-sense stationary random processes with zero means and the same covariance function $K_X(\tau)$. Let $Z(t)$ be given by

$$Z(t) = X(t) \cos \omega t + Y(t) \sin \omega t$$

Determine if $Z(t)$ is also a wide-sense stationary random process.
(i) The lifetime of a cheap light bulb is an exponential random variable with mean 36 hours. Suppose that 16 light bulbs are tested and their lifetimes measured. Use the central limit theorem to estimate the probability that the sum of the lifetimes is less than 600 hours. (Q function values that may or may not be needed to solve this problem are: $Q(0.16) \approx 0.44$, $Q(0.35) \approx 0.36$, $Q(0.90) \approx 0.18$, and an approximation to a nearest value of $y$ while using $Q(y)$ is fine.)

(ii) Suppose the random process $X(t)$ has the autocorrelation function

$$R_X(\tau) = \sigma^2 \exp(-\alpha \tau^2)$$

where $\sigma$ and $\alpha$ are constants. Explain if $X(t)$ has a mean square derivative. If yes, then find the autocorrelation function and the average power of the derivative process.
Given 10 uniformly distributed random numbers generated by MATLAB function `rand()` as follows: 0.6813, 0.3046, 0.0196, 0.8318, 0.1897, 0.3795, 0.7095, 0.4289, 0.5028, 0.8381. Use those random numbers to generate 10 random numbers with the Pareto distribution with its pdf given as \( f(x) = 2x^{-3} \).
Two variables, $u$ and $v$, are measured in an experiment. Eighteen samples are taken for each variable. The average values are: $E[u] = 5.35$, $E[v] = 4.89$; the standard deviations are: $\text{std}[u] = 1.74$, $\text{std}[v] = 1.38$; and correlation between two variables is: $R[u, v] = 0.92$. Provide an principal component analysis for the experiment.
Two random variables, $x, y$, are measured in an experiment, and we know that they have a nonlinear relationship as $y = b_0 + b_1 \sin(x)$. The following statistics are given: $\mathbb{E}[\sin(x)]=2$, $\text{Var}[\sin(x)]=4$, $\mathbb{E}[y]=3$, $\text{Var}[y]=4$, $\text{Cov}[\sin(x),y] = 2$. What are the values of $b_0, b_1$? What is the coefficient of determination ($R^2$) of the regression model?
1 Performance Analysis

Consider the following C code section:

```c
int ctr=0, i, thold=XXXX, A[100];

for(i=0; i<100; i++) {
    if (A[i] > thold)
        ctr++;
}
```

NOTE: thold represents some pre-initialized threshold.

Consider two machines, a simple accumulator machine (A), and a RISC machine (B) with assembly language versions of the code follow:

; A ; B
; Assume ctr and i are zero ; i=$s0 ctr=$s1 thold=$s2 a=$a0

```assembly
LOOP:  LOAD I  LOOP:  addi $t0, $a0, 400
        SUB ONE_HUNDRED
        JN BODY  LOOP:  beq $a0, $t0, END
        J END

BODY:  LOADI A
        SUB THOLD
        JN E N D I F

LOAD CTR ENDIF:  addi $a0, $a0, 4
ADD ONE  ENDIF:  addi $a0, $a0, 4
STORE CTR  j LOOP

ENDIF:  LOAD A
ADD ONE
STORE A

LOAD I
ADD ONE
STORE I
J LOOP
END:
```

NOTE: THERE IS NO QUESTION ON THIS PAGE.
A. Instruction Mix

Assuming that 50% of the values in the array are greater than thold, compute the instruction mix for the B by filling in the following table:

<table>
<thead>
<tr>
<th>A Instruction</th>
<th>Number</th>
<th>Percent</th>
<th>B Instruction</th>
<th>Number</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOAD</td>
<td>351</td>
<td>24.4%</td>
<td>lw</td>
<td></td>
<td></td>
</tr>
<tr>
<td>STORE</td>
<td>250</td>
<td>17.2%</td>
<td>addi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOADI</td>
<td>100</td>
<td>6.9%</td>
<td>slt</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ADD</td>
<td>250</td>
<td>17.2%</td>
<td>beq</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SUB</td>
<td>201</td>
<td>13.8%</td>
<td>j</td>
<td></td>
<td></td>
</tr>
<tr>
<td>JN</td>
<td>201</td>
<td>13.8%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JMP</td>
<td>101</td>
<td>6.7%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1454</strong></td>
<td><strong>100%</strong></td>
<td></td>
<td><strong>100%</strong></td>
<td><strong>100%</strong></td>
</tr>
</tbody>
</table>

B. CPI Calculation

Using the following table and the instruction mix you determined above, calculate the CPI for both machines.

<table>
<thead>
<tr>
<th>A Instruction</th>
<th>Number of Cycles</th>
<th>B Instruction</th>
<th>Number of Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>JMP</td>
<td>2</td>
<td>j</td>
<td>2</td>
</tr>
<tr>
<td>JN</td>
<td>2</td>
<td>bne</td>
<td>2</td>
</tr>
<tr>
<td>JZ</td>
<td>2</td>
<td>beq</td>
<td>2</td>
</tr>
<tr>
<td>LOAD</td>
<td>4</td>
<td>lw</td>
<td>5</td>
</tr>
<tr>
<td>STORE</td>
<td>3</td>
<td>sw</td>
<td>5</td>
</tr>
<tr>
<td>ADD</td>
<td>4</td>
<td>add/addi</td>
<td>4</td>
</tr>
<tr>
<td>SUB</td>
<td>4</td>
<td>sub/subi</td>
<td>4</td>
</tr>
<tr>
<td>AND</td>
<td>4</td>
<td>slt</td>
<td>4</td>
</tr>
<tr>
<td>OR</td>
<td>4</td>
<td>slt</td>
<td>4</td>
</tr>
<tr>
<td>LOADI</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>STOREI</td>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
C (d) Computers

C. MIPS Calculation Assuming that the A machine runs at 1 GHz and the B machine at 1.2 GHz, how many MIPS does each machine produce running the sample code? NOTE: Show the equations used and values substituted.

D. Execution Time What is the execution time for each program given each of the clock rates above?

E. What machine is better? Why?
2 Caches

Given a RISC processor with an 8KB direct mapped cache, 8 byte blocks, and 32-bit address space:

2.1 Bit Manipulation

Show how the address presented to the cache is divided into its components (Block, Index, and Tag). Label which field occupies which space inside the box, show the number of bits in each field below, and above label the bit numbers starting at 0 on the right and going up to 31 on the left.

```
<table>
<thead>
<tr>
<th>BIT 31</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field: (Block, Index, or Tag)</td>
</tr>
<tr>
<td>HOW MANY BITS?</td>
</tr>
</tbody>
</table>
```

2.2 Cache Line Size

What fields are required assuming that the cache described above is a write-back cache? How many bits are in each field? HINT: Draw a diagram in a format similar to the one above, providing the field names and their size in bits.
2.3 Cache Size

What is the **TOTAL** amount of information contained in the cache (in bytes)?

2.4 Memory Latency

Assuming that the given cache talks directly to memory, if a cache access is 1 clock cycle, but main memory requires 75 clock cycles, what is the average memory latency given an 80% cache hit rate?
2.5 Address Mapping

Into which index does address 42h map (assuming byte addressing)?
3 Pipelined Design

The PowerPC has more memory addressing modes than does the DLX. For example, it contains a “load and update” instruction, which takes a memory address, loads that location in memory into a register, then increments the address by a constant. For example:

```
lwu $t1, 8($s0)
```

Performs the following register transfer:

```
$t1 <- Mem[$s0], $s0 <- $s0 + 8
```

3.1 Datapath Modifications

Given the single-cycle datapath depicted above, what changes (if any) are required to implement the new instruction? **Draw your changes onto the datapath above, and explain why they are necessary below. If no changes are required, explain why below.**
Convert the datapath into a standard 5-stage RISC pipelined and explain how your modification to the datapath impacts the pipeline. Does the pipeline you proposed have to stall at any point during execution of the *lwu* instruction? If so, how can you eliminate the stall?
If \((A,B)\) is stabilizable, and there exists \(P > 0\) such that
\[ APA^T - \frac{9}{16} P + BB^T \leq 0 \]
show that \(\rho(A) < \frac{2}{3}\).
Consider the discrete-time system shown in the Figure, where

\[ M(z) = \frac{13z + 16}{20z^2} \]

where the uncertainty, \( \Delta \), is stable and \( \|\Delta\|_\infty \leq 1 \). Determine if the uncertain (closed-loop) system is stable.
Consider the nonlinear system

\[
\begin{align*}
\dot{x}_1 &= -x_1 - 6x_2 \\
\dot{x}_2 &= 3x_1 - 2x_2^3
\end{align*}
\]

Determine if

\[
V(x) = x_1^2 + 2x_2^2
\]

is a Lyapunov function for the system.
Consider the digital filter below.

(i) Suppose $x[n] = \cos(3n/5)$. Determine the smallest integer value $N > 0$, such that $y[n] = 0$ for all $n$. If no such value $N$ exists, write “$N$ does not exist for this problem.”

(ii) Suppose $x[n] = \cos(\pi n/5)$. Determine the smallest integer value $N > 0$, such that $y[n] = 0$ for all $n$. If no such value $N$ exists, write “$N$ does not exist for this problem.”

(iii) Suppose $x[n] = \cos(3\pi n/5)$. Determine the smallest integer value $N > 0$, such that $y[n] = 0$ for all $n$. If no such value $N$ exists, write “$N$ does not exist for this problem.”

(iv) Determine the frequency response, $H(e^{j\omega})$ of the digital filter (keep $N$ variable).

(v) For each value of $N$ that exists in (i)–(iii), show that $H(e^{j\omega}) = 0$ at the given input frequency thus proving $y[n] = 0$. 
Problem B

Given the impulse response

\[ h[n] = \left( \frac{1}{3} \right)^n \mu[n + 2] - (-0.5)^{n+1} \mu[-n - 1] \]

where \( \mu(n) \) is the unit step function.

a) (½ credit) Determine the transfer function \( H(z) \) and its ROC of this system. Show your work. Is the system stable? Is the system causal? Explain your reasoning.
b) (¼ credit) Determine the frequency response for this system. Write 'does not exist' if the frequency response cannot be calculated and explain how you know that it does not exist.

c) (¼ credit) Determine the impulse response for a system with identical transfer function but which is anti-causal (i.e., the impulse response is a left-sided sequence). Is this system also stable? Why or why not? Show your work clearly.
Problem C

We are interested in digitizing (sampling) an analog signal as a first step prior to digital processing. In particular, we want to preserve analog frequencies in two bands—0-10 kHz and 47-65 kHz—without any aliasing in their digitized representations. Note that we will process signals in these two bands separately, so we do not need to capture them both in the same digital signal.

a) (1/3 credit) What sampling rate do we have to use (in Hertz) to capture both bands in the same digital signal? If the quantizer in the ADC has 8 bits of precision, what will the data rate be in bits/second?

b) (1/3 credit) Assume now that we first pass the analog signal through two parallel ideal analog filters, one corresponding to each of the two bands of interest: i.e.,

These filters are designed so that all aliasing is canceled at the minimum possible sampling rates. What is the minimum sampling (in Hertz) that we can apply in each of the two channels: i.e., on $v(t)$ and $w(t)$ separately? If each of the two ADCs has 16 bits of precision, what will be the combined data rate of the two sampled signals?
c) (1/3 credit) Assume the system from part b) with the minimal sampling rates. Our goal is to notch out analog frequencies at 11.5 kHz and 55 kHz by applying digital filters to the sampled signals. What are the notch frequencies of the two digital filters (in units of radians/sample, of course)? Show your work.
In the circuit above the source is balanced, three-phase, positive sequence, 60 Hz, 4160 V. The impedance \( Z \) is a 10 ohm resistor. The line is described by the series impedance matrix given below.

\[
Z_{line} = \begin{bmatrix} 
  j2 & j1 & j1 \\
  j1 & j2 & j1 \\
  j1 & j1 & j2 \\
\end{bmatrix} \text{ ohm}
\]

Find the value of the voltage \( V_{a'b'} \)

a. Using phase domain calculations, and
b. Using symmetrical components and fault analysis principles
Energy Systems: Problem # 2

Figure 1 shows a single-line diagram of a three-phase, 60-Hz synchronous generator, connected through a transformer and parallel transmission lines to an infinite bus. All reactances are given in per-unit on a common system base.

a. If the infinite bus receives 1 per unit real power at 0.85 p.f. lagging, determine the equation for the electrical power delivered by the generator versus its power angle $\delta$.

b. The generator is initially operating in the steady-state condition when a temporary three-phase-to-ground bolted short circuit fault occurs on line 1-3 at bus 1, shown as point F in the following figure. Calculate the critical clearing time.

![Single-line diagram for transient stability analysis](image-url)
Energy Systems: Problem # 3

Figure: x

a) For the system shown in Figure x, draw positive, negative and zero sequence networks.

b) Derive bus impedance matrices for these networks: $Z_{bus}^{(0)}$, $Z_{bus}^{(1)}$, $Z_{bus}^{(2)}$.

c) Assume that a single line to ground fault occurs on phase ‘A’ at bus 2. Neglecting the pre-fault load current, find out as phasors in polar coordinates,
1. the fault current in pu,
2. fault current (pu) flowing in the three phases of the transmission line from bus#1 to bus#2.
3. voltages of the three phases at bus#1 in pu,

Note: Use of brains is more important than use of calculator. This is a short problem.
Consider a radiating aperture in an infinite planar PEC, with a given aperture E-field $\bar{E}_a(x,y)$ which has only x and y components, i.e., $\bar{E}_a = \hat{x}E_{ax} + \hat{y}E_{ay}$, see figure below. Use reciprocity (not direct application of radiation integral) and obtain an expression for the $\theta$ component (only) of the E-field, i.e., $E_{a\theta}$, at far-field for any field point represented by the spherical coordinates $(R, \theta, \phi)$ in the half space $z > 0$. The aperture is of such a size that the field there due to a distant source can be approximated as a local plane wave. Your expression should contain $E_{ax}, E_{ay}, R, \theta, \phi$, etc.
Consider a rectangular waveguide made up of 4 perfectly conducting (PEC) walls. Electric surface current $\vec{J}_x$ and magnetic surface current $\vec{M}_z$ are placed on a cross-section of the waveguide at $z = 0$, i.e., $xy$ plane, $0 \leq x \leq a$, $0 \leq y \leq b$, which will generate two waves, one propagating in $+z$ direction and one propagating in $-z$ direction.

So the fields in the waveguide are given for $z > 0$ by

$$E_x = A \sin \left(\frac{\pi y}{b}\right) e^{-j\beta z}$$

$$H_y = E_x / Z_o$$

$$H_z = \frac{A}{j\eta} \frac{f}{f} \cos \left(\frac{\pi y}{b}\right) e^{-j\beta z}$$

and the fields for $z < 0$ are given by

$$E_x = B \sin \left(\frac{\pi y}{b}\right) e^{j\beta z}$$

$$H_y = -E_x / Z_o$$

$$H_z = \frac{B}{j\eta} \frac{f}{f} \cos \left(\frac{\pi y}{b}\right) e^{j\beta z}$$

where $A$, $\beta$, $Z_o$, $\eta$, $f_c$, $f$, and $B$ are all real constants and can be considered as such in solving this problem. Suppose

$$\vec{J}_x = \hat{x} \frac{C_o}{Z_o} \sin \left(\frac{\pi y}{b}\right)$$

$$\vec{M}_z = \hat{y} C_o \sin \left(\frac{\pi y}{b}\right)$$

a) Show that $A = -C_o$ and $B = 0$.

b) Find the total time-average power flowing out of the waveguide section between $z = -l$ and $l$. 
Crossed-dipoles reside along both the x and y axes. The two dipoles are the same length, L, and the currents are given as:

\[ I_x(x') = I_o \left( 1 - \left| \frac{2x'}{L} \right| \right) \quad |x'| \leq \frac{L}{2}, \quad 0 \text{ otherwise} \]

\[ I_y(y') = jI_o \left( 1 - \left| \frac{2y'}{L} \right| \right) \quad |y'| \leq \frac{L}{2}, \quad 0 \text{ otherwise} \]

i) What are the far fields, \( E_\theta \) and \( E_\phi \)?
Suppose a flat detector is placed on the ground and it observes a Lambertian source, with radiance $L$ and area $A_s$, traveling directly overhead. Later it observes a point-like source with intensity $I$ that travels along the same overhead path as the first source. In terms of the parameters in the figure, find an expression for the ratio of the irradiance due to the Lambertian source over the irradiance due to the point source.

![Diagram of Lambertian source and point source](image)
Given a thin, positive lens of focal length \( f \) and diameter \( D \),

(a) How far from the lens must an object be placed for the image to have a transverse magnification of \( 1/2 \) ?

(b) If the image is formed a distance \( q \) behind the lens on a detector of radius \( r_{\text{det}} \), find an expression for the field-of-view of the lens/detector combination. Assume \( r_{\text{det}} < D \) and \( r_{\text{det}} < q \).

(c) For the arrangement described in (b), suppose an extended object is a distance \( p \) from the lens. The flux on the detector is \( P \). Find an expression for the radiance \( L \) of the source.
Using known transform pairs and theorems, find the Fourier transforms of the following:

(a) \( \text{sinc}(2w_1 x) \text{sinc}(2w_2 y) \)

(b) \( \text{rect}(x + a) \text{rect}(y - b) \)

(c) \( \text{comb} \left( \frac{x}{w} \right) \text{rect}(y) \)

(d) \( \text{circ} \left( \sqrt{\frac{x^2 + (y-a)^2}{w}} \right) \)

Perform the following convolutions by applying the convolution theorem:

(e) \( \text{rect} \left( \frac{x}{a} \right) \text{rect} \left( \frac{y}{b} \right) \odot \text{rect} \left( \frac{x}{a} \right) \text{rect} \left( \frac{y}{b} \right) \)

(f) \( \text{sinc} \left( \frac{x}{3} \right) \text{sinc} \left( y \right) \odot \text{sinc} \left( \frac{x}{4} \right) \text{sinc} \left( y \right) \)

Apply the central ordinate theorem to find \( \int \int g(x, y) \, dx \, dy \) for the following and compare the results with a simple area calculation:

(g) \( g(x, y) = \text{rect} \left( \frac{x}{2a} \right) \text{rect} \left( \frac{y}{2b} \right) \)
Consider two monochromatic Gaussian beams of wavelength \( \lambda \) and separated by the distance \( \Delta s \). They are shown below at the source plane \((z = 0)\).

(a) Write an expression for the optical field of one of the Gaussian beams. Assume the transverse coordinates are labeled \( x \) and \( y \), the phase of the field is zero and the beam “radius” parameter is \( w \).

(b) Write an expression that incorporates the two beams in the source plane.

(c) Find an expression for the Fraunhofer intensity pattern of the two-beam arrangement.
Consider the optical cavity shown below. Assume that the cavity is stable.

(a) Draw an equivalent lens waveguide with a unit cell. Clearly mark the unit cell starting just before the lens and proceeding in a counterclockwise direction.

(b) Derive the transmission matrix for the unit cell.

(c) Calculate the spot size and the radius of curvature of the beam on the lens if \( L = 10 \text{cm} \), \( f = 20 \text{cm} \), and the free space wavelength is 632nm.
The following graph is the measured transmission of an air-filled Fabry-Perot cavity.

(a) Find the quality factor of the cavity.

(b) Calculate the cavity length.

(c) What is the finesse?